Wave Optics

HUYGEN'S PRINCIPLE

- Every point on a wave-front may be considered a source of secondary spherical wavelets which spread out in the forward direction at the speed of light. The new wave-front is the tangential surface to all of these secondary wavelets.

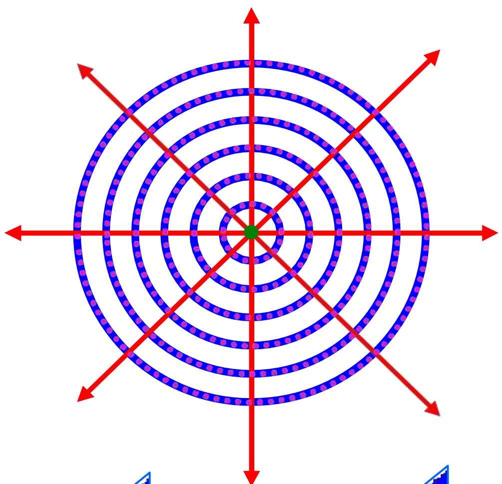
Wavelet and wavefront:

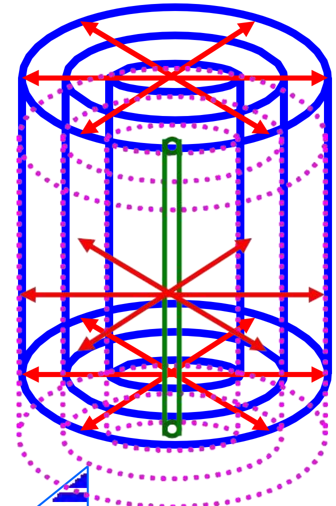
A wavelet is the point of disturbance due to propagation of light.

A wavefront is the locus of all points in space having the same phase of vibration.

A line perpendicular to a wavefront is called a 'ray'.

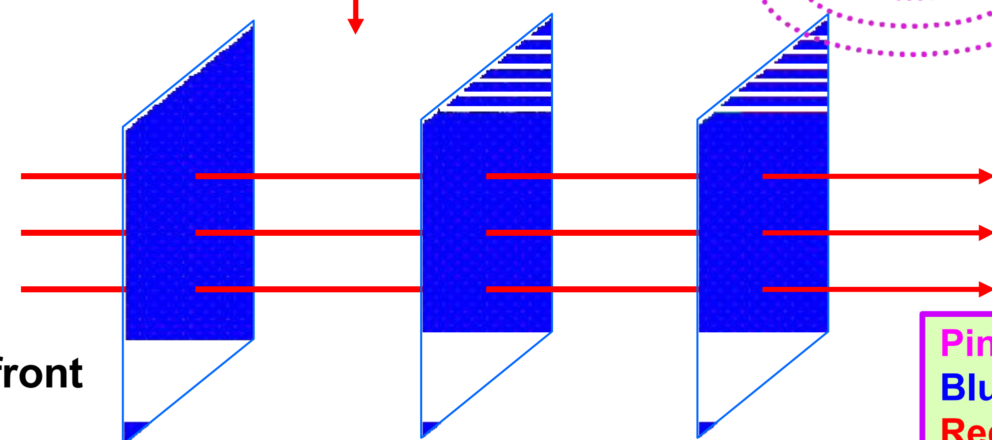
Spherical Wavefront from a point source





Cylindrical Wavefront from a linear source

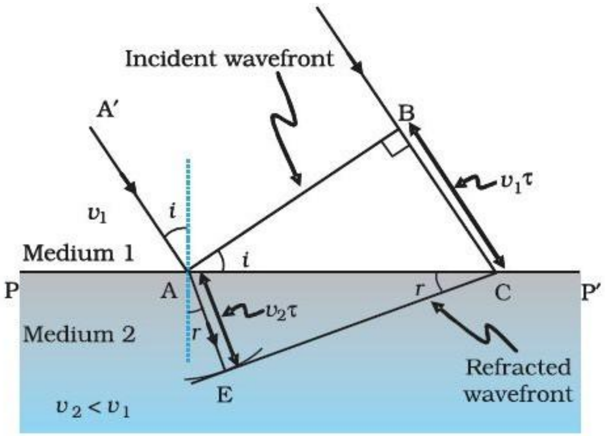
Plane Wavefront



Pink Dots – Wavelets
Blue Envelope– Wavefront
Red Line – Ray

Refraction of a plane wave

- At a denser medium



A plane wave AB is incident at an angle *i* on the surface PP′ separating medium 1 and medium 2. The plane wave undergoes refraction and CE represents the refracted wavefront. The figure corresponds to $v_2 < v_1$ so that the refracted waves bends towards the normal.

PP' - the surface separating medium 1 and medium 2

v_1- the speed of light in medium 1

v_2 – speed of light in medium 2,

AB - incident plane wavefront

EC – refracted wavefront

τ – the time taken to travel BC

BC = $v_1\tau$

AE = $v_2\tau$

$$Sin i = \frac{BC}{AC} = \frac{v_1\tau}{AC} \qquad Sin r = \frac{AE}{AC} = \frac{v_2\tau}{AC}$$

Sini/ Sinr = v_1/v_2

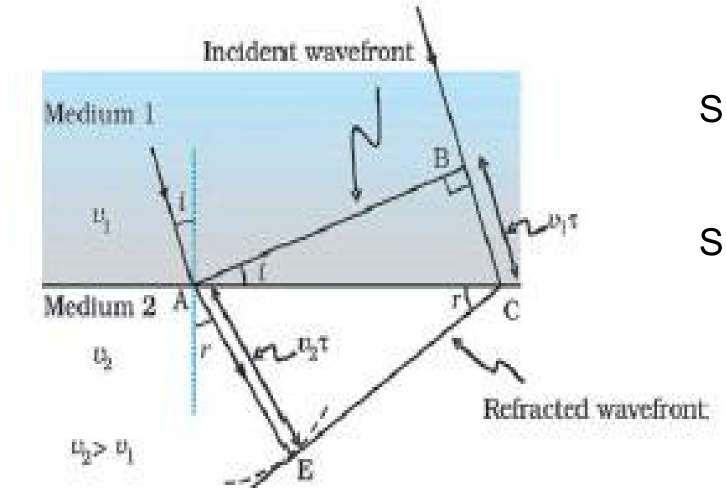
Sini/ Sinr = n_2/n_1 ⟹ **Snell's law**

n_1 – refractive index of medium 1

n_2- refractive index of medium 2

Refraction of a plane wave

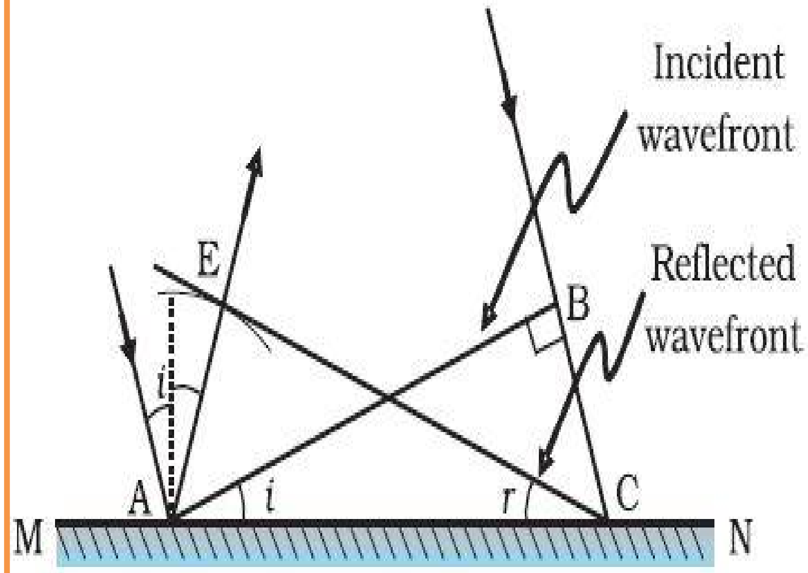
- At a rarer medium



Sini/ Sinr = v_1/v_2

Sini/ Sinr = n_2/n_1 ⟹ **Snell's law**

Reflection of a plane wave

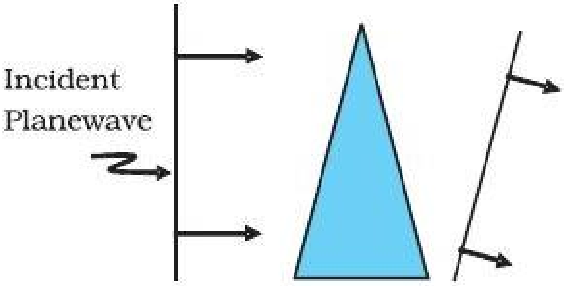


MN – reflecting surface
AB – plane incident wavfront
EC- reflected wavfront
i- angle of incidence
r- angle of reflection
v- the speed of light
τ – time taken
AE = BC = vτ
Since the triangles EAC and BAC are congruent,

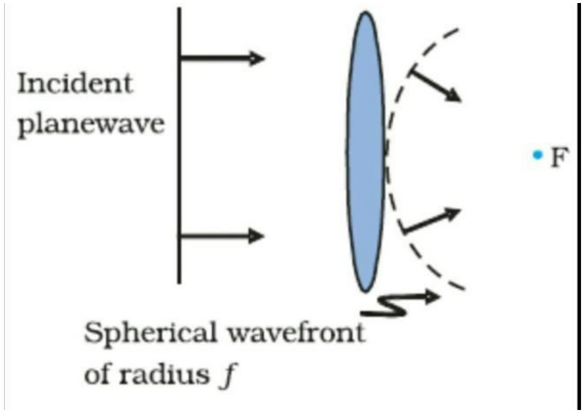
i = r , the law of reflection

Refraction of a plane wave by (a) a thin prism, (b) a convex lens. (c) Reflection of a plane wave by a concave mirror.

(a)



(b)



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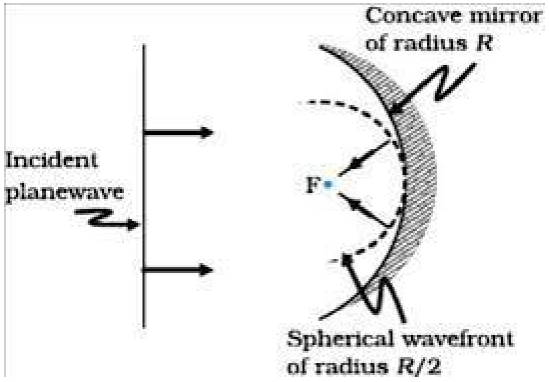


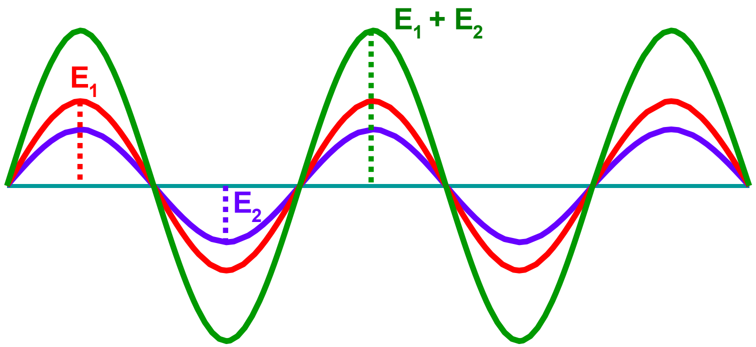
Interference

The phenomenon of one wave interfering with another and the resulting redistribution of energy in the space around the two sources of disturbance is called interference of waves.

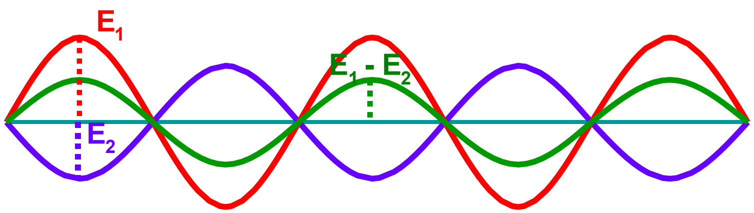
Coherent Sources

The sources emitting light waves of **same frequency** or wavelength having either a **zero or constant phase difference** are said to be coherent

Interference of Waves:

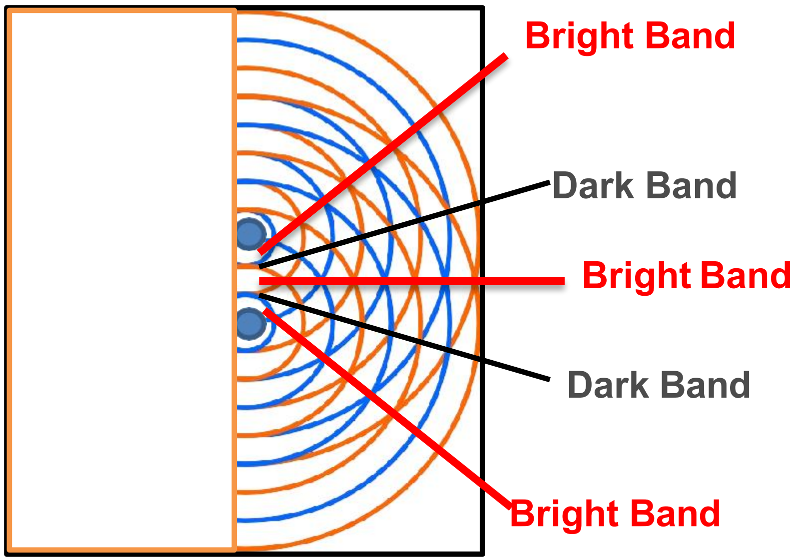


Constructive Interference $E = E_1 + E_2$



Destructive Interference $E = E_1 - E_2$

- 1st Wave (E_1)
- 2nd Wave (E_2)
- Resultant Wave
- Reference Line



- Crest
- Trough
- Bright Band
- Dark Band

The phenomenon of one wave interfering with another and the resulting redistribution of energy in the space around the two sources of disturbance is called interference of waves.

Condition for Constructive and destructive Interference

For two coherent sources vibrating in phase, constructive interference occurs when

Path difference = $n\lambda$

(n = 0, 1, 2, 3,...)

For two coherent sources vibrating in phase, destructive interference occurs when

Path difference =$(n + 1/2)\lambda$

(n = 0, 1, 2, 3,...)

$\lambda -$ wavelength

Intensity in interference

Consider two coherent light waves with intensity I_0 each and phase difference φ between them. The resultant intensity at the point of interference will be

$$I = 4I_0 \, Cos^2 (\varphi/2)$$

Condition for constructive interference : $\varphi = 0, \pm 2\pi, \pm 4\pi$.......

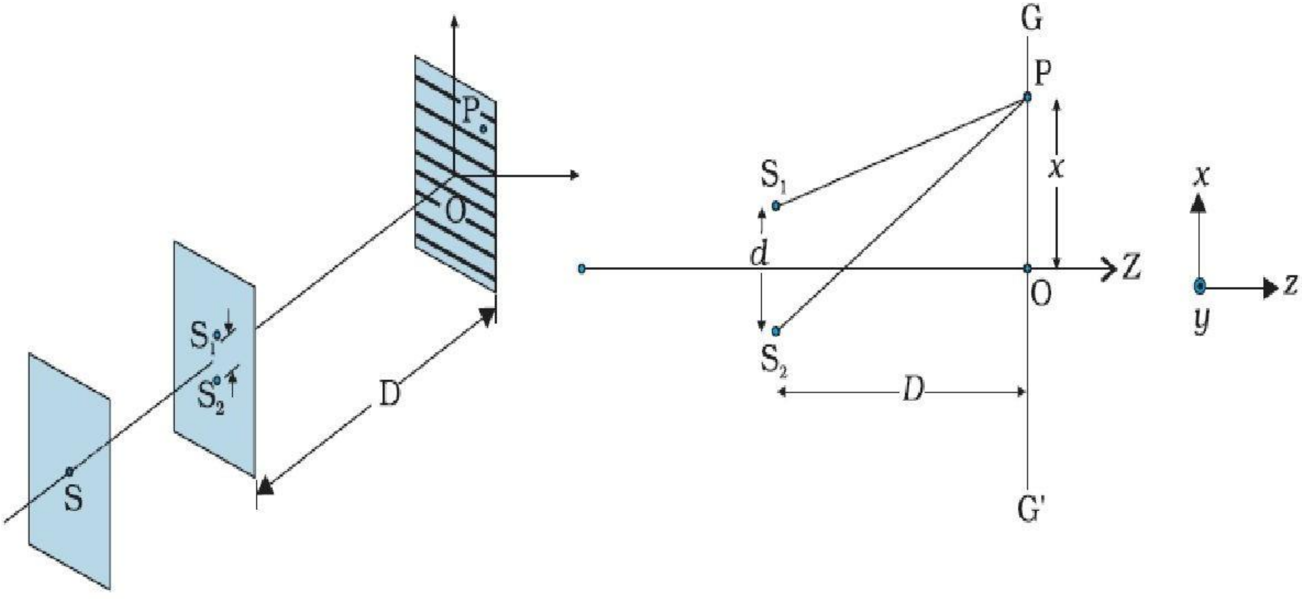
Condition for destructive interference : $\varphi = \pm\pi, \pm 3\pi$,.......

For incoherent waves the average intensity will be

$$\langle I \rangle = 4I_0 \langle Cos^2 (\varphi/2) \rangle$$

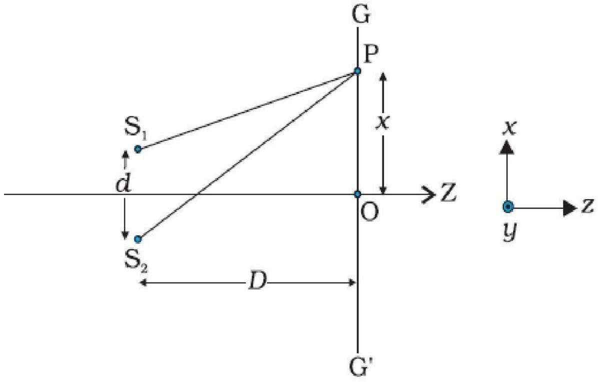
$$= 2I_0$$

Double slit experiment



(a) (b)

Path difference



The waves from S_1 and S_2 reach the point P with

Path difference $\Delta = S_2P - S_1P$

$$S_2P^2 - S_1P^2 = [D^2 + \{x + (d/2)\}^2] - [D^2 + \{x - (d/2)\}^2]$$

$$(S_2P - S_1P)(S_2P + S_1P) = 2\,x\,d$$

$$\Delta = x\,d / D$$

Positions of Bright Fringes:

For a bright fringe at P,

$\Delta = xd / D = n\lambda$

where n = 0, 1, 2, 3, …

$x = n D \lambda / d$

For n = 0, $x_0 = 0$

For n = 1, $x_1 = D \lambda / d$

For n = 2, $x_2 = 2 D \lambda / d$ ……

For n = n, $x_n = n D \lambda / d$

Positions of Dark Fringes:

For a dark fringe at P,

$\Delta = xd / D = (2n+1)\lambda/2$

where n = 0, 1, 2, 3, …

$x = (2n+1) D \lambda / 2d$

For n = 0, $x_0' = D \lambda / 2d$

For n = 1, $x_1' = 3D \lambda / 2d$

For n = 2, $x_2' = 5D \lambda / 2d$ …..

For n = n, $x_n' = (2n+1)D \lambda / 2d$

Expression for Dark Fringe Width:

$\beta_D = x_n - x_{n-1}$

$= n D \lambda / d - (n - 1) D \lambda / d$

$= D \lambda / d$

Expression for Bright Fringe Width:

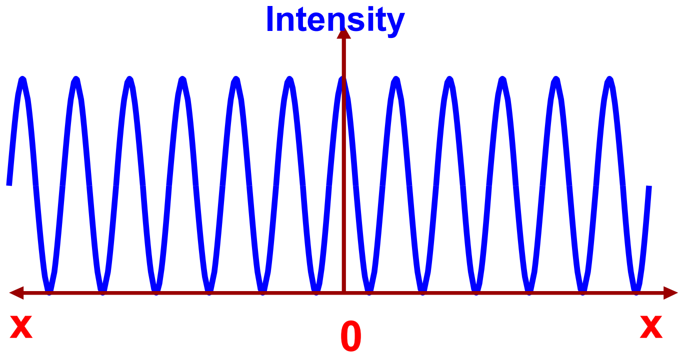
$\beta_B = x_n' - x_{n-1}'$

$= (2n+1) D \lambda / 2d - \{2(n-1)+1\} D \lambda / 2d$

$= D \lambda / d$

The expressions for fringe width show that the fringes are equally spaced on the screen.

Distribution of Intensity:



Suppose the two interfering waves have same amplitude say ‘a’, then

I_{max} α $(a+a)^2$ i.e. I_{max} α $4a^2$

All the bright fringes have this same intensity.

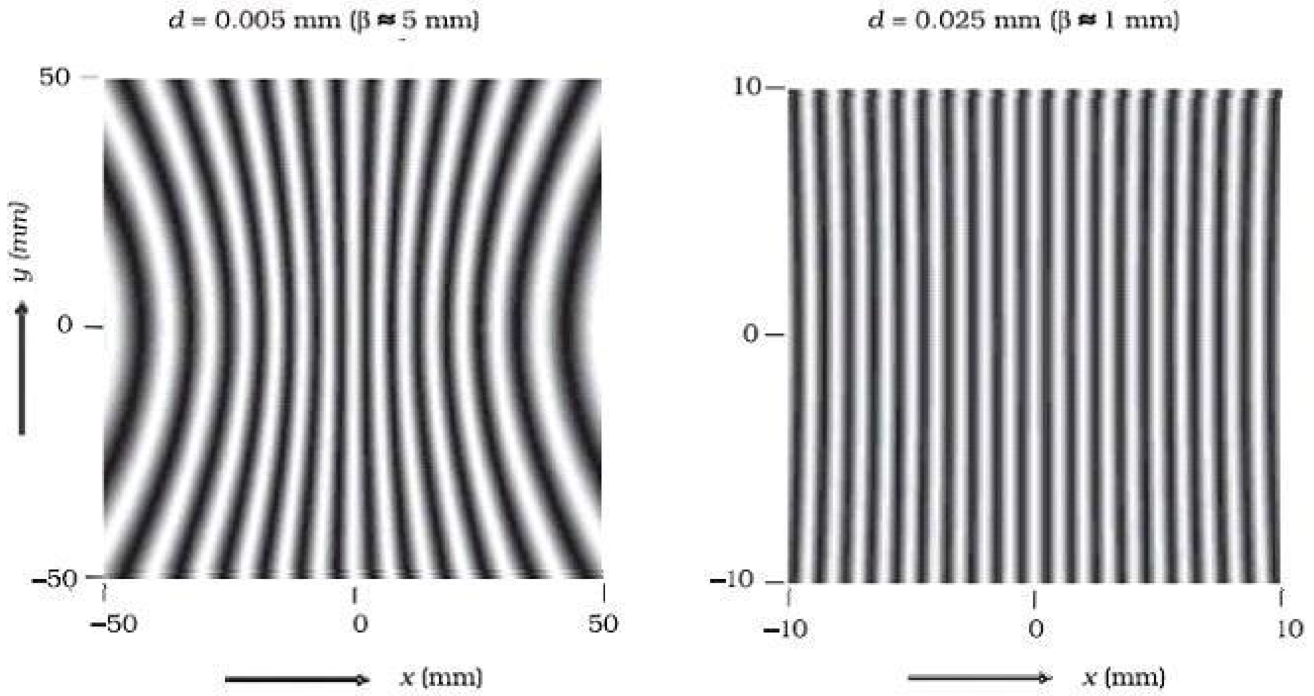
$I_{min} = 0$

All the dark fringes have zero intensity.

Conditions for sustained interference:

1. The two sources producing interference must be coherent.
2. The two interfering wave trains must have the same plane of polarisation.
3. The two sources must be very close to each other and the pattern must be observed at a larger distance to have sufficient width of the fringe. (D λ / d)
4. The sources must be monochromatic. Otherwise, the fringes of different colours will overlap.
5. The two waves must be having same amplitude for better contrast between bright and dark fringes.

A computer generated pattern



Some Examples of Interference

One of the best examples of interference is demonstrated by the light reflected from a film of oil floating on water. Another example is the thin film of a soap bubble, which reflects a spectrum of beautiful colors when illuminated by natural or artificial light sources.

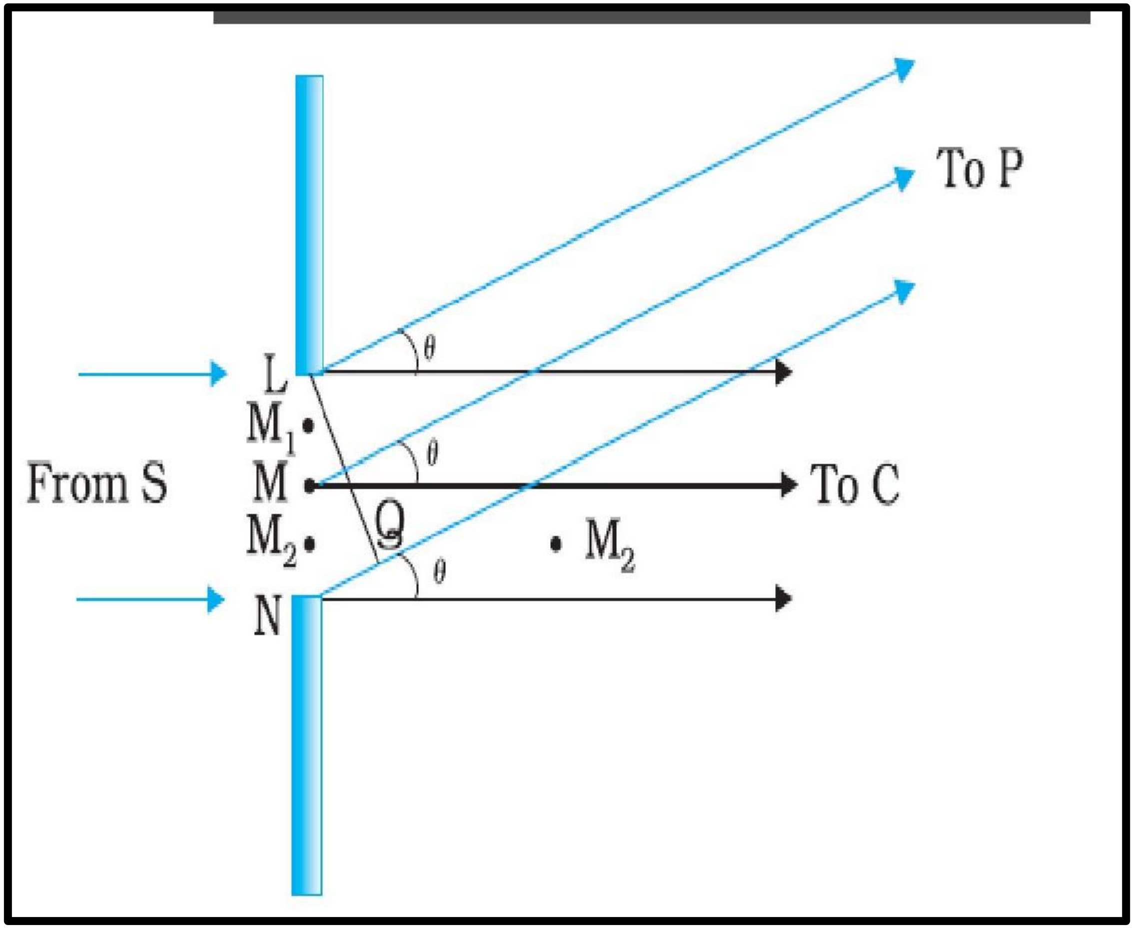
Diffraction

The phenomenon of bending of light around the sharp edges of an obstacle or aperture is called diffraction.

Condition for diffraction of light

The wavelength of incident light should be comparable to the size of the obstacle or aperture

The single slit experiment



Theory:

The path difference between the wavelets is NQ.

If 'θ' is the angle of diffraction and 'á' is the slit width, then NQ = a sin θ

To establish the condition for secondary minima, the slit is divided into 2, 4, 6, … equal parts such that corresponding wavelets from successive regions interfere with path difference of $\lambda/2$.

Or for n[th] secondary minimum, the slit can be divided into 2n equal parts.

For θ_1, $a \sin \theta_1 = \lambda$ For θ_2, $a \sin \theta_2 = 2\lambda$ For θ_n, $a \sin \theta_n = n\lambda$

Since θ_n is very small,

$a\,\theta_n = n\lambda$

$$\theta_n = n\lambda / a \quad (n = 1, 2, 3, \ldots\ldots)$$

To establish the condition for secondary maxima, the slit is divided into 3, 5, 7, … equal parts such that corresponding wavelets from alternate regions interfere with path difference of λ.

Or for n[th] secondary minimum, the slit can be divided into (2n + 1) equal parts.

For θ_1', $a \sin \theta_1' = 3\lambda/2$

For θ_2', $a \sin \theta_2' = 5\lambda/2$

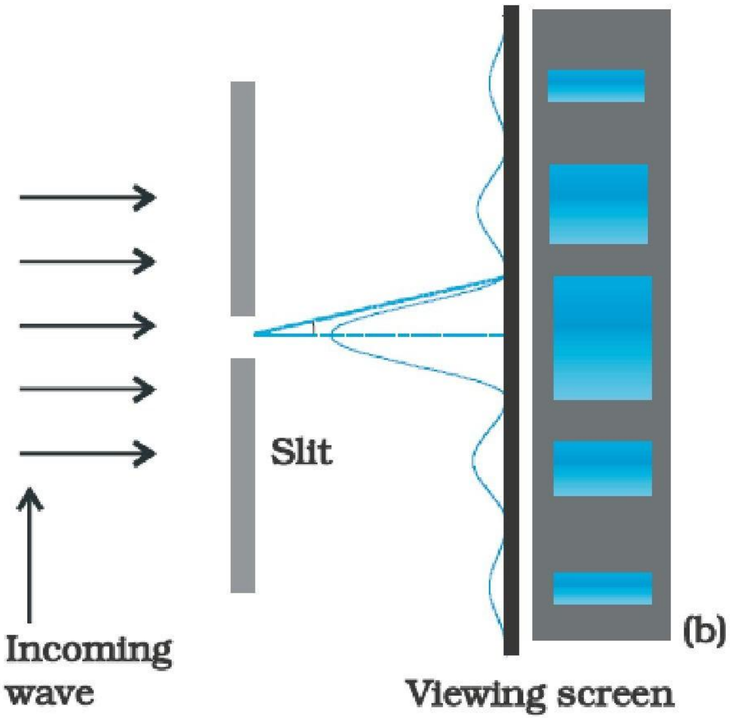
For θ_n', $a \sin \theta_n' = (2n + 1)\lambda/2$

Since θ_n' is very small,

$a\,\theta_n' = (2n + 1)\lambda / 2$

$$\theta_n' = (2n + 1)\lambda / 2a \quad (n = 1, 2, 3, \ldots\ldots)$$

Diffraction pattern

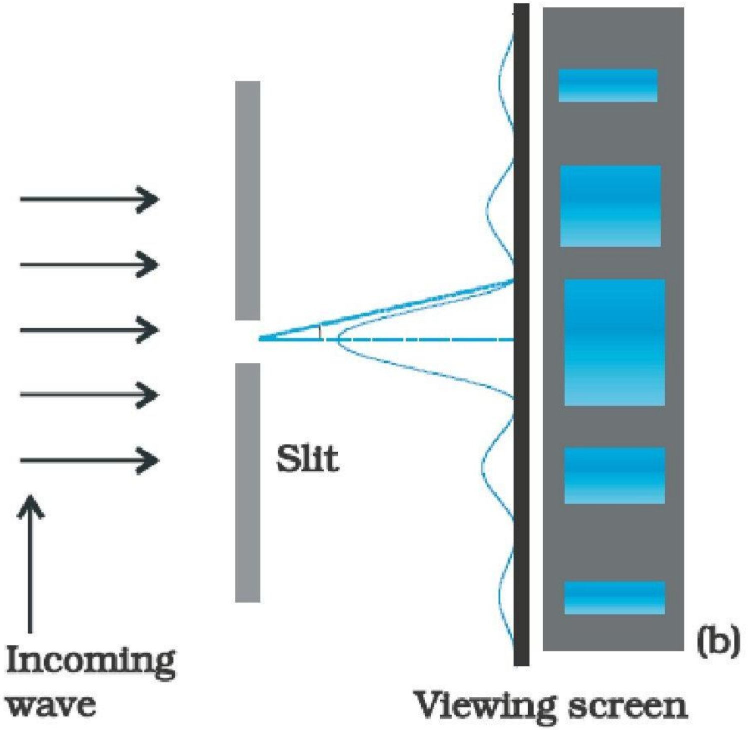


Difference between Interference and Diffraction:

Interference	Diffraction
1. Interference is due to the superposition of two different wave trains coming from coherent sources.	1. Diffraction is due to the superposition of secondary wavelets from the different parts of the same wavefront.
2. Fringe width is generally constant.	2. Fringes are of varying width.
3. All the maxima have the same intensity.	3. Central maximum has maximum intensity . The intensity falls as we go to successive maxima away from the centre on either side of central maximum

Diffraction

The pattern



Width of central maximum

The first minima on either side is at **$\theta = \lambda/a$** on either side of the central maximum

So the angular width of the central bright fringe is

$2\theta = 2\lambda/a$

The linear width of the central bright fringe is **$\beta = 2\lambda D/a$**

a- the slit width, λ- the wavelength

θ- the angular position of the first minimum from the central maximum

Fresnel's Distance:

Fresnel's distance is that distance from the slit at which the spreading of light due to diffraction becomes equal to the size of the slit.

$y_1 = D \lambda / d$

At Fresnel's distance, $y_1 = d$ **and** $D = D_F$

So, $D_F \lambda / d = d$ **or** $D_F = d^2 / \lambda$

If the distance D between the slit and the screen is less than Fresnel's distance D_F**, then the diffraction effects may be regarded as absent.**

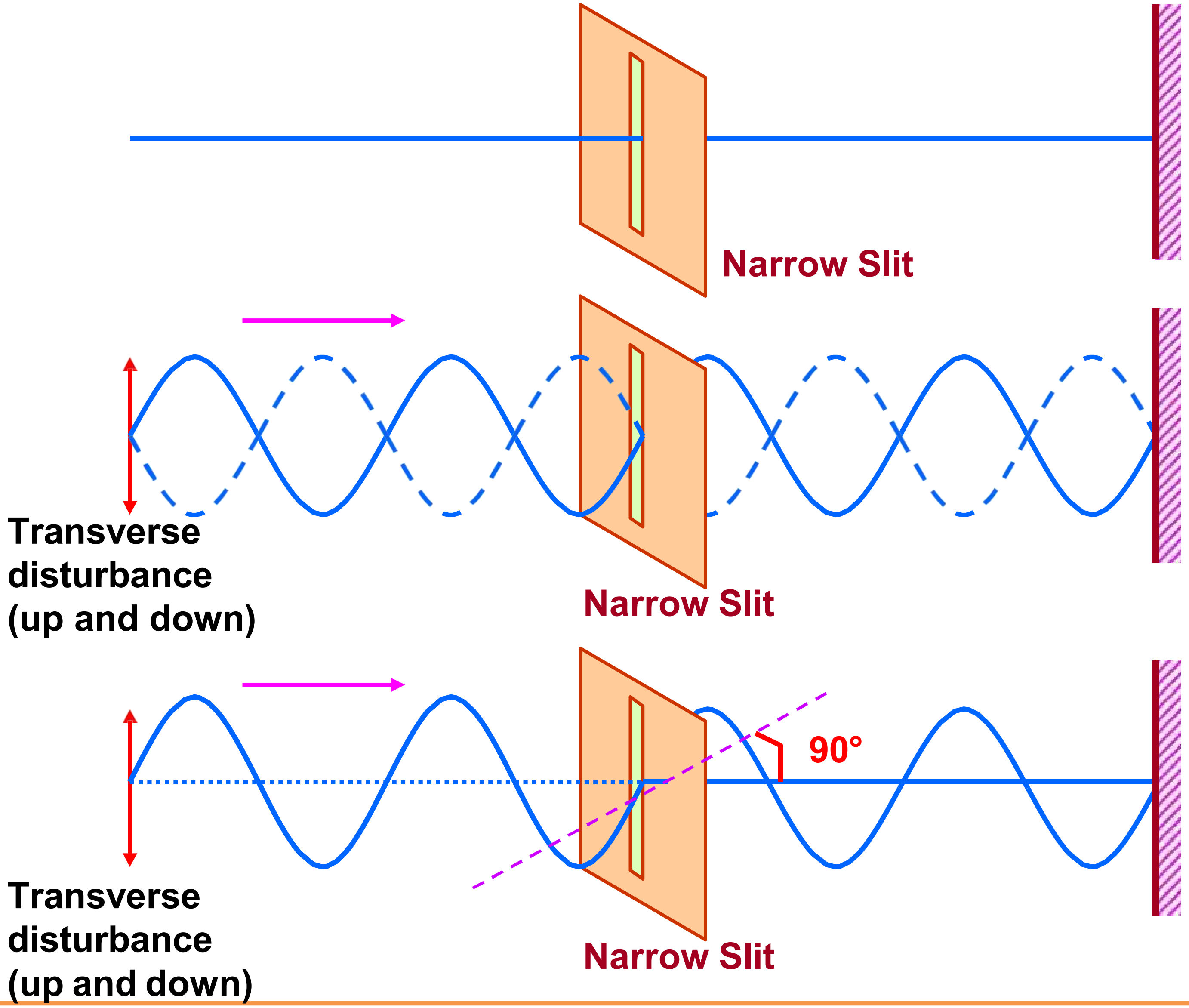
So, ray optics may be regarded as a limiting case of wave optics.

Polarisation of Light

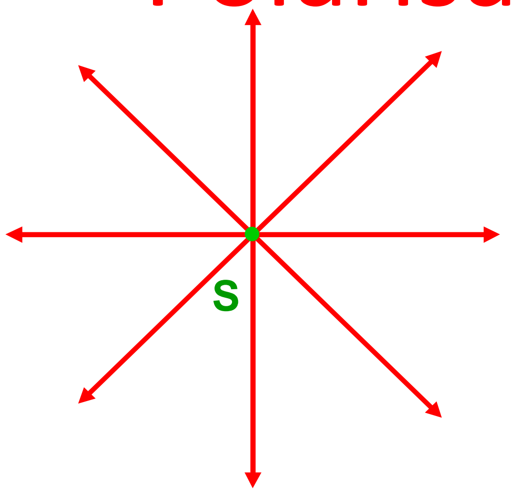
Polarisation is the phenomenon in which light or other radiation are restricted in direction of vibration

Polarized light has electric fields oscillating in one direction and **Unpolarized light** has electric fields oscillating in all directions.

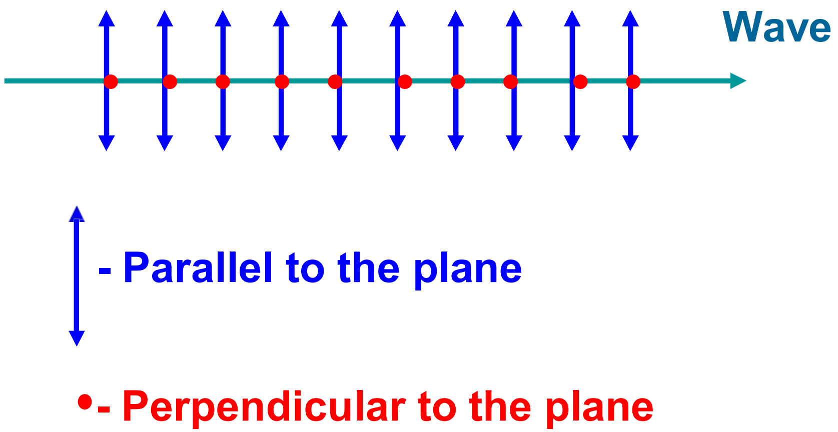
Polarisation of Transverse Mechanical Waves:



Polarisation of Light Waves:



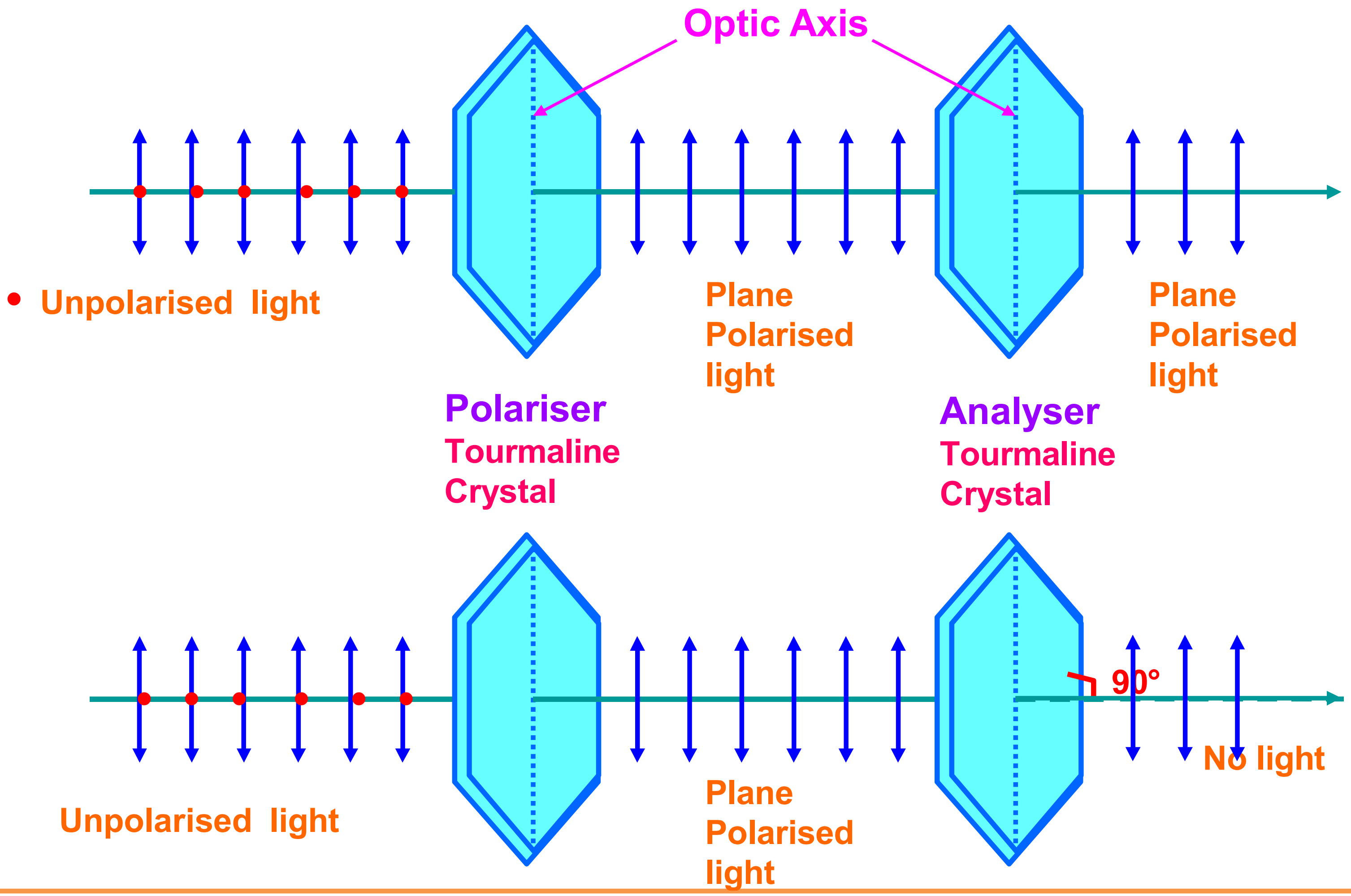
Natural Light

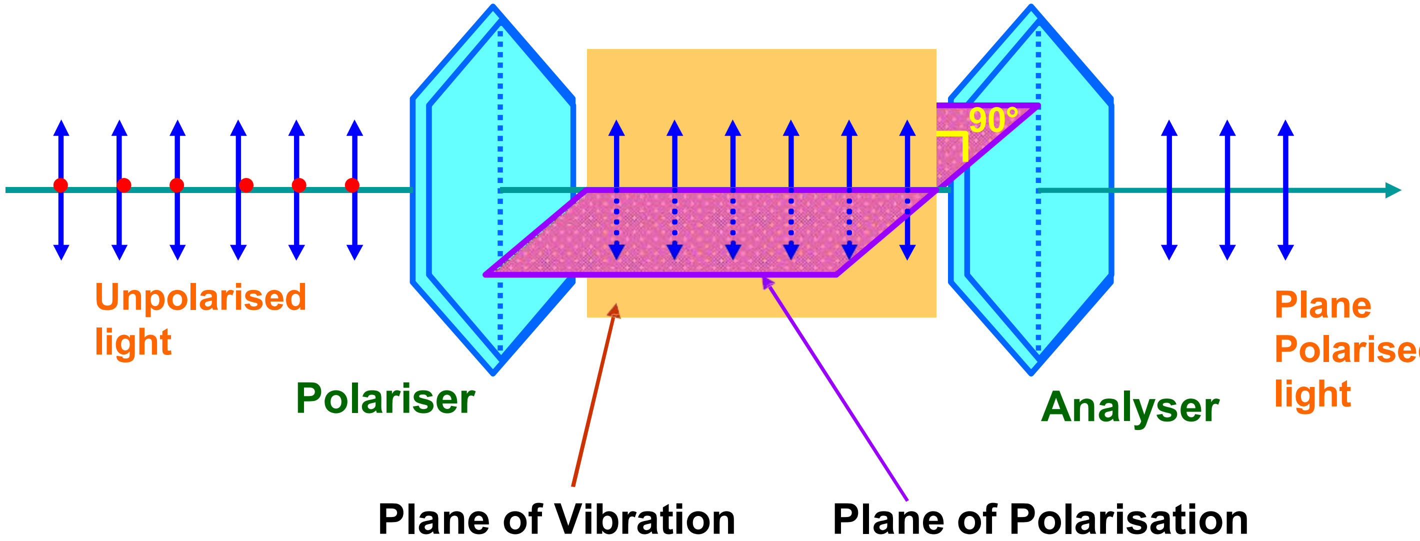


Representation of Natural Light

In natural light, millions of transverse vibrations occur in all the directions perpendicular to the direction of propagation of wave. But for convenience, we can assume the rectangular components of the vibrations with one component lying on the plane of the diagram and the other perpendicular to the plane of the diagram.

Light waves are electromagnetic waves with electric and magnetic fields oscillating at right angles to each other and also to the direction of propagation of wave. **Therefore, the light waves can be polarised.**





When unpolarised light is incident on the polariser, the vibrations parallel to the crystallographic axis are transmitted and those perpendicular to the axis are absorbed. Therefore the transmitted light is plane (linearly) polarised.

The plane which contains the crystallographic axis and vibrations transmitted from the polariser is called plane of vibration.

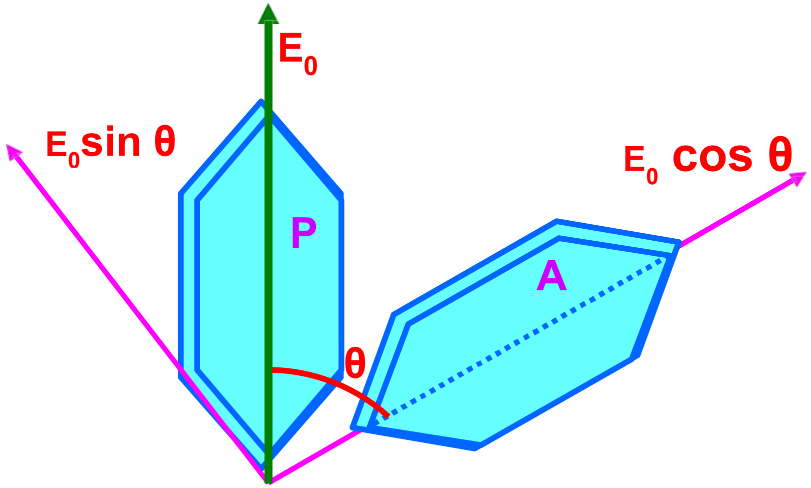
The plane which is perpendicular to the plane of vibration is called plane of polarisation.

Malus' Law:

When a beam of plane polarised light is incident on an analyser, the intensity I of light transmitted from the analyser varies directly as the square of the cosine of the angle θ between the planes of transmission of analyser and polariser.

$I \propto \cos^2\theta$

If E_0 be the amplitude of the electric vector transmitted by the polariser, then only the component $E_0 \cos\theta$ will be transmitted by the analyser.



Intensity of transmitted light from the analyser is

$$I = k\,(E_0 \cos\theta)^2$$

$$I = I_0 \cos^2\theta$$

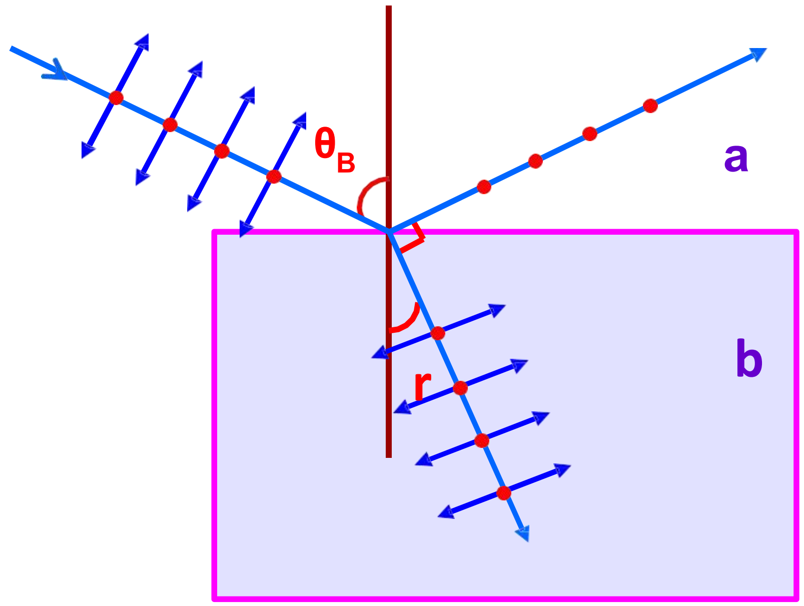
Case I : When θ = 0° or 180°, $I = I_0$

Case II : When θ = 90°, I = 0

Case III: When unpolarised light is incident on the analyser the intensity of the transmitted light is one-half of the intensity of incident light. (Since average value of $\cos^2\theta$ is ½)

Polarisation by Reflection and Brewster's Law:

When unpolarised light is incident on the boundary between two transparent media, the reflected light is polarised with its electric vector perpendicular to the plane of incidence when the refracted and reflected rays make a right angle with each other. The angle of incidence in this case is called Brewster's angle and is denoted by θ_B



$$n_{21} = \sin \theta_B / \sin r$$

$$= \sin \theta_B / \sin (90 - \theta_B)$$

$$n_{21} = \tan \theta_B$$ → Brewster's Law

Polaroids:

H – Polaroid is prepared by taking a sheet of polyvinyl alcohol (long chain polymer molecules) and subjecting to a large strain. The molecules are oriented parallel to the strain and the material becomes doubly refracting. When strained with iodine, the material behaves like a dichroic crystal.

K – Polaroid is prepared by heating a stretched polyvinyl alcohol film in the presence of HCl (an active dehydrating catalyst). When the film becomes slightly darkened, it behaves like a strong dichroic crystal.

Uses of Polaroids:

1) **Polaroid Sun Glasses**
2) **Polaroid Filters**
3) **For Laboratory Purpose**
4) **In Head-light of Automobiles**
5) **In Three – Dimensional Motion Pictures**
6) **In Window Panes**
7) **In Wind Shield in Automobiles**